Book of Abstracts

Workshop on
Spectral Geometry and Analysis of Differential Operators

Dipartimento di Matematica "Tullio Levi-Civita", Università degli Studi di Padova

September 9-11, 2019

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ISAAC

Workshop on Spectral Geometry and Analysis of Differential Operators

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Aims and Scopes

The workshop aims at gathering together experts in Spectral Theory, Spectral Geometry, Analysis of Partial Differential Operators, Homogenization and Asymptotic Analysis.

The focus is on the interplay between the spectral properties of partial differential operators and the geometry of the underlying system. In particular the following topics will be discussed: spectral stability, shape optimization and extremum properties for eigenvalues, universal and asymptotic estimates for eigenvalues, upper and lower bounds, functional and isoperimetric inequalities, thin and perforated domains, boundary homogenization.

The workshop aims at stimulating new collaborations, contaminations, and the developing of new ideas and strategies, as well as to present new challenging problems.

The organizing committee: P.D. Lamberti, P. Musolino, L. Provenzano.

Contents

Invited talks

Thin domains with oscillatory boundaries

José M. ARRIETA

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We will present some results on the behavior of the Laplace operator with Neumann boundary condition in thin domains having boundaries with an oscillatory behavior. We will obtain the homogenized limit in some relevant cases including the periodic and locally periodic situation. Moreover, we will include recent results dealing with some higher dimensional thin domains with a quasiperiodic oscillatory behavior of the boundaries.

Based on joint work with Manuel Villanueva-Pesqueira.

Keywords: thin domain, oscillatory boundaries, homogenization.

Best Sobolev constants in the presence of sharp Hardy terms in Euclidean and hyperbolic space

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In this article we compute the best Sobolev constants for various Hardy-Sobolev inequalities with sharp Hardy term. This is carried out in three different environments: interior point singularity in Euclidean space, interior point singularity in hyperbolic space and boundary point singularity in Euclidean domains.

Joint work with A. Tertikas.

Minimal k-partition for the p-norm of the eigenvalues

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In this talk,we would like to analyze the connections between the nodal domains of the eigenfunctions of the Dirichlet-Laplacian and the partitions of the domain by k open sets D_i which are minimal in the sense that the maximum over the D_i's of the groundstate energy of the Dirichlet realization of the Laplacian is minimal. Instead of considering the maximum among the first eigenvalues, we can also consider the p-norm of the vector composed by the first eigenvalues of each subdomain.

Keywords: Minimal partitions; shape optimization; Dirichlet-Laplacian eigenvalues

On spectral stability problem for a pair of self-adjoint elliptic differential operators on bounded open sets

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The Dirichlet eigenvalue problem will be under discussion:

$$Hu = \lambda Mu \quad \text{on} \quad \Omega,$$

where H and M are self-adjoint elliptic differential operators of orders $2k$, $2m$ respectively, where $k, m \in \mathbb{N}, m < k$, and Ω is an arbitrary bounded open set in $\mathbb{R}^N$.

Sharp estimates for the variation of the eigenvalues upon domain perturbation will be presented.

This work was supported by the grants of the Russian Science Foundation (project no. 19-11-00087) and of the Russian Foundation for Basic Research (project no. 18-51-06005).

Based on joint work with Tamara Tararykova and Bien Thanh Tuyen.

Keywords: eigenvalues, elliptic differential operators, perturbation of domain.

Norm resolvent convergence for perturbed planar waveguides

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We consider an elliptic operator in a planar infinite strip perturbed in different way:

- by a perforation by small holes along a curve:

  we impose mixed classical boundary conditions (Dirichlet, Neumann and Robin) on the holes, assuming that the perforation is non-periodic and satisfies rather weak assumptions.

- by substituting one side of the boundary by a fast oscillating curve:

  we assume that both the period and the amplitude of the oscillations are small and impose the Dirichlet condition on the upper boundary and Dirichlet, Neumann or Robin boundary condition on the oscillating boundary.

- by infinite numbers of "windows":

  we impose the Dirichlet condition on the upper boundary and frequent alternation boundary condition on the lower boundary. The alternation is introduced by the periodic partition of the boundary into small segments on which Dirichlet and Neumann (the "windows") conditions are imposed in turns.

In all cases we describe the homogenized operators, establish the norm resolvent convergence of the perturbed resolvents to the homogenized one, prove the estimates for the rate of convergence and study the convergence of the spectrum.
Based on joint work with [D. Borisov].

Keywords: homogenization, norm-resolvent convergence, spectrum .

Bounds for the first eigenvalue of the magnetic Laplacian on domains

Bruno COLBOIS

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We consider the magnetic Laplacian with curvature 0 on bounded, smooth domains of the Euclidean plane, and with the magnetic Neumann boundary conditions. We find lower and upper bounds for the first eigenvalue depending on the geometry and on the topology of the domain. We also discuss the sharpness of these estimates.

Based on joint work (in progress) with Alessandro Savo.

Keywords: Magnetic Laplacian, First eigenvalue, Bounds.

Some questions of shape optimization arising from additive manufacturing

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Metal additive manufacturing is an industrial revolution since it offers the possibility to produce complex parts without the design constraints of traditional manufacturing routes. As it is well suited to the production of small series, it becomes mandatory in the aeronautic industry. It offers the possibility to realize extremely complex shapes presenting different scales. As a consequence, this technology generates a lot of nice mathematical questions coupling multiscale analysis and shape optimization.

I will present some ongoing work on these questions in particular for shape filled with lattices. I will focuss on how to take into account the skin effect, on the simultaneous design of the shape of the object and of the material properties and on some method to achieve robustness of the design.

Based on joint work with F. Caubet (Pau) and H. Harbrecht (Basel).

Keywords: shape optimization, asymptotic analysis, additive manufacturing.

On eigenvalues of the fractional Laplacian under removal of small capacity sets

Veronica FELLI

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In this talk we consider the eigenvalue problem for the restricted fractional Laplacian in a bounded domain with homogeneous Dirichlet boundary conditions. We introduce the notion of fractional capacity for compact subsets, with the property that the eigenvalues are not affected by the removal of zero fractional capacity sets. Given a simple eigenvalue, we remove from the domain a family of compact sets which are concentrating to a set of zero fractional capacity and we detect the asymptotic expansion of the eigenvalue variation; this expansion depends on the eigenfunction associated to the limit eigenvalue. The case in which the family of compact sets is concentrating to a singleton will be described in details.

Based on joint work with Laura Abatangelo and Benedetta Noris.

Keywords: Fractional Laplacian, Asymptotics of eigenvalues. Fractional capacity.

On a class of fourth order Steklov eigenvalue problems

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We talk about some spectral stability results for a class of fourth order Steklov eigenvalue problems with respect to domain perturbation. In the first part of the talk, we explain what we mean by fourth order Steklov eigenvalue problems, we describe their physical interpretation and we show their relevant role in the study of comparison principles for the bilaplacian. The central part of the talk is devoted to the main stability results obtained in a recent work published in Calc. Var. PDE in 2019. We find sufficient conditions for the validity of the spectral stability with respect to domain perturbation and we show their optimality. Finally, we present some results about stability and instability for a suitable "Navier-to-Neumann map" and for a class of Navier-type problems.

Based on a joint work with Pier Domenico Lamberti.

Keywords: Biharmonic operators, Steklov boundary conditions, spectral stability.

Robin eigenvalues on domains with cusps

Hynek KOVARIK

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Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$, be a bounded domain with an outward power-like cusp which is assumed not too sharp in a suitable sense. We consider the Laplacian $u \mapsto -\Delta u$ in Ω with the Robin boundary condition $\partial_n u = \alpha u$ on $\partial\Omega$ with ∂_n being the outward normal derivative and $\alpha > 0$ being a parameter. We show that for large α the associated eigenvalues $E_j(\alpha)$ behave as $E_j(\alpha) \sim -\epsilon_j \alpha^\nu$, where $\nu > 2$ and $\epsilon_j > 0$ depend on the dimension and the cusp geometry. This is in contrast with the well-known estimate $E_j(\alpha) = O(\alpha^2)$ for the Lipschitz domains.

Based on a joint work with Konstantin Pankrashkin.

Keywords: Robin Laplacian, asymptotic expansion, domains with cusps.

Rellich inequalities via Hardy inequalities on manifolds and graphs

Yehuda PINCHOVER

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We establish weighted Rellich-type inequalities with best constants for Schrödinger operators defined either on noncompact Riemannian manifolds or graphs. The corresponding weights satisfy certain eikonal inequality involving Hardy-weights.

Based on joint works with M. Frass, and B. Devyver (2014), and M. Keller and F. Pogorzelski (2019).

Keywords: Agmon metric, Eikonal inequality, positive solutions.

Heat content, exit time moments and isoparametric foliations

Alessandro SAVO

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We consider two functionals on a smooth compact domain in a Riemannian manifold : the first is the heat content at time t, and the second is the k-th exit time moment for any given positive integer k. The scope is to classify those domains which are critical for either functionals (at all times t and, respectively, for all k). We show in particular that criticality for any of the two functionals is equivalent to the existence of an isoparametric foliation of the domain, that is, a foliation by parallel, constant mean curvature hypersurfaces having only one (minimal) singular leaf.

Keywords: Heat content, Isoparametric hypersurfaces, Moment spectrum.

New spectral bounds for damped systems

Christiane TRETTER

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In this talk we present enclosures for the spectra of operators associated with second order Cauchy problems for the case of non-selfadjoint damping. These new results yield much better bounds than the numerical range for both uniformly accretive and sectorial damping, and even in the case of selfadjoint damping. Applications e.g. to wave equations illustrate the results.

Joint work with Birgit Jacob, Carsten Trunk and Hendrik Vogt.

Sharp estimates for solutions to Elliptic Problems with Robin boundary conditions

Cristina TROMBETTI

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Comparison results (of isoperimetric type) for Elliptic Problems with Dirichlet and Neumann boundary conditions are well established results since decades. So far not many results have been obtained in the case of Robin boundary conditions and in this talk, we shall investigate some open questions related to a Faber Krahn inequality and Talenti-type estimates.

Based on joint work with [Angelo Alvino & Carlo Nitsch].

Keywords: Robin boundary conditions, Eigenvalues, Symmetrization.

Contributed talks

Semiclassical bounds for spectra of biharmonic operators

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We consider Riesz means of the eigenvalues of the biharmonic operator subject to various boundary conditions, and we improve the known sharp semiclassical bounds in terms of the volume of the domain with a second term with the expected power. We obtain such estimates by means of the averaged variational principle. This method intrinsically also yields two-sided bounds for individual eigenvalues, which are semiclassically sharp.

Based on a joint work with L. Provenzano and J. Stubbe.

Keywords: Biharmonic operator, Riesz means, eigenvalue asymptotics, semiclassical bounds for eigenvalues.

Global hyperbolic propagators

Matteo CAPOFERRI

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In my talk I will present a global, invariant and explicit construction of hyperbolic propagators on closed Riemannian manifolds. This can be achieved by representing the propagator as a single Fourier integral operator — global both in space and in time — with distinguished complex-valued phase function. The knowledge of the propagator allows one, in turn, to recover asymptotic spectral properties of the operators at hand. The main focus of the talk will be on the wave propagator. Time permitting, I will outline similarities and fundamental differences between scalar equations and first order systems, such as, e.g., the massless Dirac equation and Maxwell's equations.

Based on joint work with D. Vassiliev (UCL) and M. Levitin (Reading).

Keywords: Global Fourier integral operators, hyperbolic propagators, Weyl coefficients.

Some sharp spectral inequalities

Giuseppina DI BLASIO

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We prove upper and lower bounds for Dirichlet eigenvalues of linear and nonlinear operators

Based on joint work with [F. Della Pietra and N. Gavitone].

Keywords: Efficiency, Dirichlet eigenvalues, optimal estimates.

Geometric inequalities through p-capacitary potentials

Mattia FOGAGNOLO

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We show that the level set flow of p-capacitary potential of a domain can be employed to deduce sharp geometric inequalities. The main technical tool is a monotone quantity along this level set flow, first provided for the electrostatic potential of a domain in $\mathbb{R}^n$ by V. Agostiniani and L. Mazzieri. This new strategy is very flexible, and we are going to see that leads to new sharp Willmore-type and Minkowksi-type inequalities in relevant families of noncompact Riemannian manifolds with nonnegative Ricci curvature. A combination of our inequalities with the Mean Curvature Flow also provides new Isoperimetric Inequalities on these manifolds.

Based on joint works with V. Agostiniani, L. Benatti, L. Mazzieri and A. Pinamonti.

Keywords: geometric inequalities, p-Laplacian, Ricci curvature.

Hardy inequalities in the Heisenberg group

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We present recent results about Hardy inequalities in the n-th Heisenberg group. We show that, contrary to the Euclidean case, a radial Hardy inequality, i.e., a Hardy inequality taking into account only the directional derivative w.r.t. the sub-Riemannian distance, does not hold in this context for any dimension. Motivated by this fact, we then suggest the study of a non-radial Hardy inequality, based on the construction of specific polar-type coordinates following from the explicit synthesis of sub-Riemannian geodesics. We prove a sharp weighted non-radial inequality that imply (non-sharp) bounds for the non-radial Hardy constant on homogeneous cones. We underly through the latter a strong difference with respect to the Euclidean case.

Based on joint work with Dario Prandi.

Keywords: Hardy inequality, Heisenberg group, sub-Riemannian distance.

Courant-sharp Robin eigenvalues for the square

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Let S be a square in $\mathbb{R}^2$ of side-length π. Consider the eigenfunctions of the Dirichlet Laplacian acting in $L^2(S)$ that achieve equality in Courant's Nodal Domain theorem. These eigenfunctions and their corresponding eigenvalues are called Courant-sharp. We also consider the corresponding Courant-sharp Neumann eigenvalues of S.

A result due to Pleijel (1956) asserts that the Courant-sharp Dirichlet eigenvalues of the square are the first, second and fourth (a complete proof of this result was given by Bérard and Helffer in 2015). Helffer and Persson-Sundqvist (2015) proved that the Courant-sharp Neumann eigenvalues of the square are the first, second, fourth, fifth and ninth.

The Robin eigenvalues of the Laplacian with positive parameter interpolate between the Neumann eigenvalues and the Dirichlet eigenvalues. We discuss whether the Robin eigenvalues of the square are Courant-sharp when the Robin parameter is large.

Based on joint work with Bernard Helffer, Université de Nantes.

Keywords: Courant-sharp, Robin eigenvalues, square.

On asymptotic behaviour of solutions of the Dirac system

Alexander GOMILKO

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The main focus of this talk is the following matrix Cauchy problem for the Dirac system on the interval $[0,1]$:

$$D'(x)+\begin{bmatrix} 0 & \sigma_1(x) \\ \sigma_2(x) & 0 \end{bmatrix} D(x)=i\mu \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} D(x), \quad D(0)=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

where $\mu \in \mathbb{C}$ is a spectral parameter, and $\sigma_j \in L_2[0,1]$, $j=1,2$. We propose a new approach for the study of asymptotic behaviour of its solutions as $\mu \to \infty$ and $|\operatorname{Im}\mu| \le c$. As an application, we obtain new, sharp asymptotic formulas for eigenfunctions of Sturm–Liouville operators with singular potentials:

$$\begin{gathered} y''(x)+q(x)y(x)=\lambda y(x), \quad x\in[0,1], \\ y(0)=y(1)=0, \end{gathered}$$

where a potential $q(x)=\sigma'(x)$, and $\sigma \in L_2[0,1]$.

The talk is based on joint work with Łukasz Rzepnicki, accepted for publications in *Journal of Spectral Theory* (arXiv:1808.09272).

Keywords: Dirac system, Sturm–Liouville problem, singular potential.

From Steklov to Neumann via homogenisation at the critical regime

Jean LAGACÉ

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In the homogenisation of boundary value problems, there is often a critical regime where a phase transition can be observed in the limiting problem. This prompted Ciaronescu and Murat to call this phenomenon "A strange term coming from nowhere". For the Steklov problem, we show that that the homogenised limit at the critical regime is a dynamical eigenvalue problem studied by J. von Below and G. François. A distinct feature of this problem is that the eigenvalue appears both in the interior problem, as in traditional eigenvalue problems, and on the boundary, as is the case for the Steklov problem. We will then see how we can recover universal bounds for the normalised Neumann eigenvalues in terms of bounds for the Steklov eigenvalues from the study of this dynamical problem.

Based on joint work with Alexandre Girouard (Laval) and Antoine Henrot (Nancy).

Keywords: Steklov problem, homogenisation, eigenvalue bounds.

Multiple Steklov eigenvalues in a domain with a small hole

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Let Ω^o be a bounded open domain of $\mathbb{R}^n$. Let ν_{Ω^o} denote the outward unit normal to $\partial\Omega^o$. We assume that the Steklov problem $\Delta u = 0$ in Ω^o, $\frac{\partial u}{\partial \nu_{\Omega^o}} = \lambda u$ on $\partial\Omega^o$ has a multiple eigenvalue $\tilde{\lambda}$ of multiplicity r. Then we consider an annular domain $\Omega(\epsilon)$ obtained by removing from Ω^o a small cavity of size $\epsilon > 0$, and we show that under appropriate assumptions each elementary symmetric function of r eigenvalues of the Steklov problem $\Delta u = 0$ in $\Omega(\epsilon)$, $\frac{\partial u}{\partial \nu_{\Omega(\epsilon)}} = \lambda u$ on $\partial\Omega(\epsilon)$ which converge to $\tilde{\lambda}$ as ϵ tends to zero, equals real a analytic function defined in an open neighborhood of $(0,0)$ in $\mathbb{R}^2$ and computed at the point $(\epsilon, \delta_{2,n}\epsilon\log\epsilon)$ for $\epsilon > 0$ small enough. Here $\nu_{\Omega(\epsilon)}$ denotes the outward unit normal to $\partial\Omega(\epsilon)$, and $\delta_{2,2} \equiv 1$ and $\delta_{2,n} \equiv 0$ if $n \geq 3$. Such a result is an extension to multiple eigenvalues of a previous result obtained for simple eigenvalues in collaboration with S. Gryshchuk.

Keywords: Multiple Steklov eigenvalues and eigenfunctions, singularly perturbed domain, Laplace operator, real analytic continuation.

Invisible inclusions and a spectral $\mathbb{R}$-linear problem

Vladimir MITYUSHEV

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An eigenvalue $\mathbb{R}$-linear problem arisen in the theory of invisible and neutral inclusions is discussed by a method of integral equations. Consider non-overlapping simply connected domains D_k $(k = 1, 2, \cdots, n)$ in the unit disk U. We find functions $\varphi_k(z)$ analytic in D_k, $\varphi(z)$ in $D = U \setminus \cup_{k=1}^{n} (D_k \cup \partial D_k)$ and $\varphi_0(z)$ in $|z| > 1$, respectively, and find a complex constant $\lambda \neq 0$ such that the following $\mathbb{R}$-linear conditions are fulfilled

$$\varphi(t) = \varphi_k(t) - \rho_k \overline{\varphi_k(t)}, \quad t \in \partial D_k, \quad k = 1, 2, \ldots, n, \tag{1}$$

$$\varphi(t) = \overline{\lambda} \varphi_0(t) - \overline{\varphi_0(t)}, \quad |t| = 1, \quad \varphi_0(\infty) = 0. \tag{2}$$

Here, the constants $|\rho_k| < 1$ are given. A nodal domains conjecture on the eigenfunction $\varphi_0(z)$ is posed. Demonstration of the conjecture allows to justify that a set of inclusions can be made invisible by surrounding it with an appropriate coating.

Based on joint work with Natalia Rylko.

Keywords: "$\mathbb{R}$-linear spectral problem" "invisible inclusions" "neutral inclusion"

Eigenvalues of the Laplacian with moving mixed boundary conditions: sharp asymptotics in terms of boundary capacity

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In this talk I will consider the eigenvalue problem for the Laplace operator with Neumann boundary conditions and a perturbation of it, which consists in imposing Dirichlet boundary conditions in a small subset of the boundary. In this framework, I will state the sharp asymptotic behaviour of a perturbed eigenvalue in the case in which it is converging to a simple eigenvalue of the Neumann problem. The first term in the asymptotic expansion turns out to depend on the "capacity" of the subset where the perturbed eigenfunction is vanishing. Finally I will focus on the case of Dirichlet boundary conditions imposed on a subset which is scaling to a point.

Based on joint work with Veronica Felli and Benedetta Noris.

Keywords: mixed eigenvalue problem, Sobolev capacity, asymptotics of eigenvalues.

The Steklov problem on graphs

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Recently the Steklov eigenvalues of a graph have been introduced. They are defined analogously to the eigenvalues of the Steklov problem on a Euclidean domain. This discretization process raises many questions about the Steklov eigenvalues on graphs that are inspired by spectral geometry. In this talk, I will discuss the discrete Steklov problem and present lower bounds for the first non-zero eigenvalue.

Keywords: Discrete Steklov operator.

Bounded H_∞-calculus for differential operators on conic manifolds with boundary

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We shall discuss the existence of a bounded H_∞-calculus for realizations (closed extensions) of differential operators on manifolds with conical singularities and with boundary, subject to differential boundary conditions. The existence is guaranteed by posing suitable conditions of parameter-ellipticity, the proof relies on techniques from pseudodifferential operators on singular manifolds.

Based on joint work with N. Roidos and E. Schrohe.

Keywords: Bounded H_∞-calculus, conic manifolds with boundary, parameter-dependent pseudodifferential operators.

Spectral estimates of nonlinear elliptic operators in non-convex domains

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We study the Neumann eigenvalue problem for the nonlinear p-Laplace operator:

$$-\mathrm{div}(|\nabla u|^{p-2}\nabla u) = \mu_p |u|^{p-2}u \text{ in } \Omega, \ \frac{\partial u}{\partial n} = 0 \text{ on } \partial\Omega,$$

in bounded domains $\Omega \subset \mathbb{R}^2$ that satisfy the quasihyperbolic boundary condition. On the base of the geometric theory of composition operators on Sobolev spaces we give spectral estimates of the first non-trivial Neumann eigenvalue $\mu_p(\Omega)$ in the terms of the (quasi)conformal geometry of domains.

Based on joint work with [Vladimir Gol'dshtein and Valerii Pchelintsev].

Keywords: Elliptic operators, Sobolev spaces, quasiconformal mappings.

Shape optimization problems on doubly connected domains

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The problem of finding a domain, in a given class of domains, which optimizes the first nonzero eigenvalue of certain eigenvalue problem has been extensively studied in the literature. In this talk, we discuss about the following similar problems:

1. For $n > 2$, let B_1 and B_2 be two open balls in $\mathbb{R}^n$ of fixed radius such that $\overline{B}_1 \subset B_2$. Consider the eigenvalue problem

$$\begin{array}{rcll} \Delta\varphi & = & 0 & \text{in } B_2 \setminus \overline{B}_1, \\ \varphi & = & 0 & \text{on } \partial B_1, \\ \frac{\partial\varphi}{\partial\nu} & = & \tau\varphi & \text{on } \partial B_2, \end{array} \tag{3}$$

   where ν is the outward unit normal to ∂B_2. Then the first eigenvalue of (3) attains its maximum if and only if B_1 and B_2 are concentric.

2. Let $(\mathbb{M}, ds^2)$ be a non-compact rank-1 symmetric space. Let $B_0 \subset M$ be a geodesic ball centered at a point $p \in M$, and $D \subset M$ be a domain of fixed volume such that $D = \exp_p(N_0)$, where N_0 is a symmetric neighborhood of the origin in T_pM and $\overline{B}_0 \subset D$. Consider the following problem

$$\begin{array}{rcll} \Delta\varphi & = & \mu\varphi & \text{in } D \setminus \overline{B}_0, \\ \frac{\partial\varphi}{\partial\nu} & = & 0 & \text{on } \partial\left(D \setminus \overline{B}_0\right), \end{array} \tag{4}$$

   where ν is the outward unit normal to $\partial\left(D \setminus \overline{B}_0\right)$. Then the first nonzero eigenvalue of (4) attains its maximum if and only if D is a geodesic ball centered at p.

Based on joint work with Prof. G. Santhanam.

Keywords: The mixed Steklov-Dirichlet eigenvalue problem, Neumann eigenvalue problem, Variational characterization.